Diagonalization Principle

Let S be a non-empty set and R any relation on S.

Let

 $D = \{a \in A \mid (a,a) \in r\}$

For each $a\epsilon A$, let $R_a = \{ b | (a,b) \epsilon R \}$

Then diagonalization principle states that D is different from each R_a .

OR

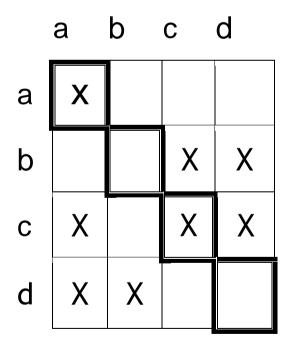
Diagonalization principle states that the complement of the diagonal is different

from each row

For example,

Let S = {a, b, c, d} R = { (a,a), (b,c), (b,d), (c,a), (c,c), (c,d), (d,a), (d,b) }

The above relation R is shown in matrix from as follows:

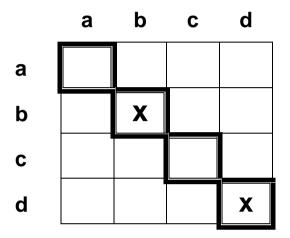


Diagonal elements are marked.

From the figure,

$$R_a = \{a\}$$

$$\begin{split} &\mathsf{R}_{b} = \{c,d\} \\ &\mathsf{R}_{c} = \{a,\,c,\,d\} \\ &\mathsf{R}_{d} = \{a,\,b\} \\ &\mathsf{Diagonal}\;\mathsf{D} = \{a,b\} \\ &\mathsf{Complement}\;\mathsf{of}\;\mathsf{Diagonal},\;\mathsf{D}^{{}^{{}}} = \{b,d\} \end{split}$$



If we compare each of the above R_a , R_b , R_c and R_d with D, we can see that D is different from each.

Thus complement of the diagonal is distinct from each row.

Note:

Following information is needed to solve the example given below:

9's complement of a number 9's complement of 276 is 723.

9's complement of 425 is 574.

9's complement of 793 is 206.

Example 1:

Prove that the set of real numbers between 0 and 1 is uncountable.

An example for a real number between 0 and 1 is 0.34276

Let us represent a real number between 0 and 1 as

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\mathsf{X} = \mathsf{0.X}_0\mathsf{X}_1\mathsf{X}_2\mathsf{X}_3....
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where each x_i is a decimal digit.

Let f(k) be an arbitrary function from natural numbers to the set [0,1]. We can arrange the elements in a 2d array as,

f(0):	:	X_{00}	X_{01}	X ₀₂	X ₀₃	_	_	—				
f(1):	:	X ₁₀	X ₁₁	X ₁₂	X ₁₃	_	_	_				
f(2):	:	X ₂₀	X ₂₁	X ₂₂	X ₂₃	_	_	_				
		-										
f(n):	:	X _{n0}	X _{n1}	X _{n2}	X _{n3}	_	_	_				

where x_{ni} is the ith digit in the decimal expansion of f(n).

Next we find the complement of the diagonal (9's complement) as follows: $Y = .y_0y_1y_2$ where $y_i = 9$'s complement of x_{ii}

(Find out the 9's complement of x_{00} , x_{11} , x_{22} , x_{33} ... x_{nn}

From the diagonalisation principle, it is clear that the complement of the diagonal is different from each row.

Here it is clear that Y is different from each f(i) in at least one digit. $Y \neq f(i)$.

Hence Y cannot be present in the above array.

This means that the set of real numbers between 0 and 1 is countably infinite or not countable.

as,

For instance suppose	we	arr	ange	e the	real numbers					
f(0):		9	4	2	4					
f(1):		6	3	6	2					
f(2):		2	8	6	4					
f(3)	•	6	5	3	2					
f(n):		4	1	5	7					

Here the diagonal is 9362

The complement (9's complement) of the diagonal is 0 6 3 7

The real number .0637... is not in the above table.

Thus It is clear that this value is distinct from each row, f(0), f(1), f(2),...f(n).